

# High School Geometry – Unit 1

## Develop the ideas of congruence through constructions and transformations

**Critical Area**: In this Unit the notion of two-dimensional shapes as part of a generic plane (the Euclidean Plane) and exploration of transformations of this plane as a way to determine whether two shapes are congruent or similar are formalized. Students use transformations to prove geometric theorems. The definition of congruence in terms of rigid motions provides a broad understanding of this notion, and students explore the consequences of this definition in terms of congruence criteria and proofs of geometric theorems. Students develop the ideas of congruence and similarity through transformations.

CLUSTERS	COMMON CORE STATE STANDARDS
Make geometric construction Make a variety of formal geometric constructions using a variety of tools.	<ul> <li>Geometry - Congruence</li> <li>G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software etc. Copying a segment, copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines including the perpendicular bisector of a line segment; and constructing a line parallel to a give line through a point not on the line.</li> <li>G.CO.13 Construct an equilateral triangle, a square, a regular hexagon inscribed in a circle.</li> </ul>
Experiment with transformations in the plan Develop precise definitions of geometric figures based on the undefined notions of point, line, distance along a line and distance around a circular arc. Experiment with transformations in the plane.	<ul> <li>Geometry - Congruence</li> <li>G.CO.1 Know precise definitions of angle, circle, perpendicular lines, parallel lines, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.</li> <li>G.CO.2 Represent transformations in the plane using e.g. transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g. translation versus horizontal stretch.)</li> <li>G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.</li> <li>G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles perpendicular lines, parallel lines, and line segments.</li> <li>G.CO.5 Given a geometric figure and a rotation, reflection or translation, draw the transformed figure using e.g. graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.</li> </ul>
Understand congruence in terms of rigid motions	Geometry - Congruence
Use rigid motion to map corresponding parts of congruent triangle onto each other. Explain triangle congruence in terms of rigid	<b>G.CO.6</b> Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
motions.	G.CO.7 Use definition of congruence in terms of rigid motions to show that two triangles are congruent if

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CLUSTERS	COMMON CORE STATE STANDARDS	
	and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	
	<b>G.CO.8</b> Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow the definition of congruence in terms of rigid motions.	
<b>Prove geometric theorems</b> Prove theorems about lines and angles, triangles; and parallelograms.	<ul> <li>Geometry - Congruence</li> <li>G.CO.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.</li> <li>G.CO.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides</li> </ul>	
	<ul><li>G.CO.11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent; the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.</li></ul>	
MATHEMATICAL PRACTICES		
<ol> <li>Make sense of problems and persevere in solving them.</li> <li>Reason abstractly and quantitatively.</li> <li>Construct viable arguments and critique the reasoning of others.</li> <li>Model with mathematics.</li> <li>Use appropriate tools strategically.</li> <li>Attend to precision.</li> <li>Look for and make use of structure.</li> <li>Look for and express regularity in repeated reasoning.</li> </ol>	As you begin the year, it is advised that you start with MP1 and MP 3 to set your expectations of your classroom. This will help you and your students become proficient in the use of these practices. All other practices may be evident based on tasks and classroom activities.	
	LEARNING PROGRESSIONS	

(m)Major Clusters – area of intensive focus where students need fluent understanding and application of the core concepts.

(s)Supporting/Additional Clusters – designed to support and strengthen areas of major emphasis/expose students to other subjects.

 $\star$ Indicates a modeling standard linking mathematics to everyday life, work, and decision-making.

(+) Indicates additional mathematics to prepare students for advanced courses.

	ENDURING UNDERSTANDINGS		ESSENTIAL QUESTIONS		KEY VOCABULARY
•	The fundamental tools of classic construction are the	•	How do geometric constructions relate to	•	alternate Interior Angles

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	ENDURING UNDERSTANDINGS		ESSENTIAL QUESTIONS		KEY VOCABULARY
	compass and the straightedge but, there are many		geometric to geometric reasoning and proof?	•	compass
	other tools useful for constructions including; string,			•	congruence
	reflective devices, protractor, and geometric	•	What are the justifications that can be used to	•	construction
•	Software. Geometric construction is a visual representation of		guide geometric constructions?	•	corresponding
•	geometric principals and develops a deeper		What are the ariteria that can be used by a	•	distance
	understanding of the spatial relationships between		geometry student to select the most appropriate	•	equilateral Triangle
	pairs of figures and their elements.		tools and software for geometric constructions?		mapping
•	Transformations include a variety of motions that		8	•	mapping
	take a set of points in the plane as input and gives us	•	What are the similarities and differences among	•	midpoint
	other points as output.		the various transformations and how can they be	•	non-rigid motion
•	There are rigid transformations that preserve		grouped as either rigid or non-rigid?	•	parallel Lines
•	distance and angles and non-rigid transformations			•	parallelogram
	that do not.	•	How can the properties of rigid motion be used to	•	perpendicular Lines
			SAS SSS)?	•	protractor
٠	The properties of transformations that are rigid		516,555).	•	reflection
	motion can be used to identify and prove	•	What are the various pathways to create a valid	•	rigid Motion
	congruence of figures in a plane.		proof for theorems about lines, angles, triangles		straightedge
•	Constructing a viable argument using the precise		congruence and parallelograms?		transformations
•	vocabulary of transformations and congruence to			•	translation
	prove geometric theorems in a variety of formats is			•	vertical Angles
	important to Geometry proof.				6

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
LAUSD Adopted Textbooks and Programs	Engage students to investigate more closely the definition that	Formative Assessment
Big Ideas Learning - Houghton Mifflin Harcourt 2015: Big Ideas Geometry	shapes are congruent when they "have the same size and	PARCC -
<u>College Preparatory Mathematics, 2013: Core</u> <u>Connections, Geometry</u>	the plane, but now, students build more precise definitions for	http://www.parcconline.org/samples/ma
<ul> <li><u>The College Board, 2014:Springboard</u> <u>Geometry</u></li> </ul>	the rigid motions (rotation, reflection, and translation) based on previously defined and understood terms, such as point, line, between, angle, circle, perpendicular, etc. (G-CO.1,3,4).	<u>thematics/high-school-mathematics</u> <u>http://www.parcconline.org/sites/parcc/f</u> ilos/BARCC_SamplaItams_Mathematic
Materials: For Students: compass, protractor, straight-edge, string, reflective devices, tracing paper, graph paper and geometric software.	Help students strengthen their understanding of these definitions by transforming figures using patty paper,	s_HSGeoMathIIIGeometricConnection 081913_Final_0.pdf

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
	transparencies, or geometry software, (G-CO.2, 3,5, MP.).	
For instruction: Document camera, LCD projector,	Transformations should be investigated both in a general plane	
screen	as well as on a coordinate system especially when explicitly	LAUSD Assessments
Websites: <u>Math Open Reference</u> <u>http://mathopenref.com/tocs/constructionstoc.html</u> (online resource that illustrates how to generate constructions)	<ul> <li>describing transformations using precise names of points, translation vectors, and lines of symmetry or reflection.</li> <li>Concrete Models – Students make use of visual tools for representing geometric figures, such as simple patty paper or transparencies, graph paper, calculators, reflective devices.</li> </ul>	The district will be using the SMARTER Balanced Interim Assessments. Teachers would use the Interim Assessment Blocks (IAB) to monitor the progress of students. Each IAB can be given twice to show growth over time
Math is Fun	dynamic geometry software, or other manipulatives as they	
http://www.mathsisfun.com/geometry/constructions.	work through transformations. Have students show using rigid	
<u>html</u> H-G.CO.12, 13	motions that congruent triangles have congruent corresponding	
	parts, and that, conversely, if the corresponding parts of two	State Assessments
Manga High	triangles are congruent, then there is a sequence of rigid	California will be administering the
http://www.mangahigh.com/en_us/games/transtar	motions that takes one triangle to the other. For example:	SMARTER Balance Assessment as the
Engage New York http://www.engageny.org/sites/default/files/resource /attachments/geometry-m1-teacher-materials.pdf		end of course for grades 3-8 and 11. There is no assessment for Algebra 1. The 11th grade assessment will include ítems from Algebra 1, Geometry, and Algebra 2 standards. For examples, visit
	A D	the SMARTER Balance Assessment at: http://www.smarterbalanced.org/ Sample Smarter Balanced Items:
	Illustration of the reasoning that corresponding parts being	http://sampleitems.smarterbalanced.org/it
	congruent implies triangle congruence, in which point A is	empreview/sbac/index.ntm
	translated to D, the resulting image of $\Delta$ ABC is rotated so as to	
	place B onto E, and finally, the image is then reflected along	
	line segment DE to match point C to F.	
	Geometry Construction – Students use a variety of tools and	
	methods to make formal geometric constructions, such as:	
	copying a segment; copying an angle; bisecting a segment;	
	bisecting an angle; constructing perpendicular lines, including	
	the perpendicular bisector of a line segment; and constructing a	

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
	line parallel to a given line through a point not on the line.	
	Teachers should use a variety of strategies for engaging	
	students in understanding and writing proofs, including: using	
	ample pictures to demonstrate results and generate strategies;	
	using patty paper, transparencies, or dynamic geometry	
	software to explore the steps in a proof; creating flow charts	
	and other organizational diagrams for outlining a proof; and	
	writing step-by-step or paragraph formats for the completed	
	proof (MP.5).	

### LANGUAGE GOALS for low achieving, high achieving, students with disabilities and English Language Learners

- Students justify congruency statements using key vocabulary, such as: mapping, translation, reflection, rotation, rigid motion, and congruence.
- Students describe their understanding of a construction using key vocabulary, such as: bisect an angle, perpendicular bisector, and parallel lines.
- Students identify words in word problems that help them formulate arguments and evaluate arguments to make specific claims about congruence; they will use the sentence starter, "The words \_\_\_\_\_\_ and \_\_\_\_\_ lead me to believe..."
- Students compare two geometric shapes using comparative adjectives.
- Students will compare transformations in the plane and describe their changes using academic language and complete sentences.

#### PERFORMANCE TASKS

Circles in Triangles <u>http://map.mathshell.org/materials/tasks.php?taskid=256#task256</u> http://map.mathshell.org/materials/tasks.php?taskid=258#task258

#### DIFFERENTIATION ACCELERATION **UDL/FRONT LOADING INTERVENTION** Model and review constructions (online Know the basic properties of the different types of Students can learn to prove and develop theorems for triangles (equilateral, equiangular, isosceles, right transformations that are not on the coordinate plane using resources). angle, scalene, obtuse, acute). conditional statements in their explanations. Include and use vocabulary lists with visual Work with construction tools: drawing circles, Condense the units of circles and transformations aids. measuring with compass, drawing lines. together; use the properties of circles to determine points Use heterogeneous groups for peer assistance Know how to name angles, points, lines, rays, of rotation. and modeling. FRONT LOADING ACCELERATION **INTERVENTION** segments and length. Combine dilations and similarity, showing parallelism, angle congruence in dilated figures and the definition of Know distance, midpoint and slope formulae. Know how to plot points. dilation to prove shapes are similar through AA. Make use of isosceles triangle and third angles theorems.

### **References:**

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- 2. McCallum, W., Zimba, J., Daro, P. (2011, December 26 Draft). *Progressions for the Common Core State Standards in Mathematics*. Cathy Kessel (Ed.). Retrieved from <a href="http://ime.math.arizona.edu/progressions/#committee">http://ime.math.arizona.edu/progressions/#committee</a>.
- 3. Engage NY. (2012). New York Common Core Mathematics Curriculum. Retrieved from <a href="http://www.engageny.org/resource/high-school-geometry">http://www.engageny.org/resource/high-school-geometry</a>.
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- 9. The University of Arizona. (2011-12). Progressions Documents for the Common Core Math Standards. Retrieved from <a href="http://ime.math.arizona.edu/progressions">http://ime.math.arizona.edu/progressions</a>.

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